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# Probabilistic Approach to Damage Tolerance Design of Aircraft Composite Structures

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**Many probabilistic methodologies have been proposed that address the reliability and risk factors unique to composite structures, incorporating micromechanics, laminate theory, manufacturing defects, operating environment, and impact damage, but few have addressed the importance of inspection intervals and damage-detection capabilities. The present study is based on a probabilistic damage-tolerance analysis with consideration of the following parameters: inspection intervals; statistical data on damages; loads; temperatures; damage-detection capability; and residual strength of the new, damaged, and repaired structures. The inspection intervals are formulated based on the damaged structure's probability of failure and the quality of its repair. This approach enables accidental, random damage events to be assessed quantitatively, allowing aircraft manufacturers, operators, and flight certification authorities to better evaluate and predict the damage tolerance and safety of an aircraft structure. Engineers can use this methodology to incorporate structural risk and maintenance costs into their design and inspection criteria. Its validity is demonstrated on several existing structural components, and special attention has been paid to the availability and cost of the probabilistic input data.**

## Nomenclature

| | | |
|---|---|---|
| $b$, $B$ | = | exponential probability density function parameter |
| $C_V$ | = | coefficient of variation |
| $F_{l\max}(x, t)$ | = | cumulative probability distribution function (cumulative distribution function) of maximum load $x$ per time $t$ |
| $G$ | = | residual strength curve parameter |
| $H(x)$ | = | frequency of an event exceeding the level $x$ |
| $N_D$ | = | number of damages per life |
| $P_D(D)$ | = | probability of detecting a damage/defect with a size greater than $D$ |
| $P_f$, POF | = | probability of failure |
| $S_i$ | = | strength through $i$th interval |
| $T$ | = | temperature, K |
| $t_i$ | = | time interval |
| $\alpha$ | = | shape parameter |
| $\beta$ | = | scale parameter |
| $\Phi_0$ | = | tabulated Gauss–Laplace function (normal distribution) |

## Introduction

RISK analysis methods for determining aircraft reliability have been under development for more than 30 yr and, because many key engineering parameters are probabilistic in nature (material property values, gust load, damage size, and so on), research has focused on the development of probabilistic methods. Despite their advantages over classical, deterministic safety factors in the design process, design organizations have been reluctant to adopt even the standard probabilistic methods or to include them as part of their risk analysis capability. Reasons cited include: the complexity of failure modes; lack of available damage data; and safety issues.

Current, deterministic, approaches to aircraft design and flight certification use safety and knockdown factors for various design conditions (e.g., moisture, temperature, loading and damage). Because of the higher scatter in statistical data for composite materials and their sensitivity to impact damage, traditional methods have led to very conservative design and service guidelines. Such methods, essentially, assume that a worst case scenario occurs simultaneously for each design condition. The result is a substantial weight and cost penalty, reducing the performance advantages of composites [1].

The need to quantify the reliability of aircraft structures subject to accidental damage is an increasingly important issue as the use of composite materials becomes more widespread. Foreign object damage (FOD), ground vehicle collisions and lightening strikes are but a few examples of accidental damage that an aircraft structure must face during its operational lifetime. A probabilistic approach enables these accidental, random damage events to be assessed quantitatively, allowing aircraft manufacturers, operators, and flight certification authorities to better evaluate and predict the damage tolerance and safety of an aircraft structure.

The primary candidate for addressing this issue has been the software package, NESSUS, developed by the Southwest Research Institute (SwRI) [2–4]. However its advanced fast probability integration (FPI) methods do not work with discrete variables, such as the number of damage events or the number of inspections necessary to detect damage. Other candidates are the "Level of Safety" method proposed by Lin, et al. [5] and the "Probabilistic Design of Composite Structures" (ProDeCompos) method proposed by Styuart, et al. [6]. The approach presented here combines these last two methods, and uses the NESSUS package to obtain the probabilistic characteristics of initial and residual strength and to validate the proposed method. Our method incorporates data from in-service experience to optimize the benefits of using composites, but its potential to increase structural safety and reduce maintenance cost is not limited to composite structures.

Past studies of structural risk and reliability have focused on metal fatigue in aging aircraft. Composite structures, however, are fatigue and corrosion resistant, but because of their brittle behavior during failure, composite structures are much more sensitive to impact damage. Furthermore, there may be no visible evidence even when significant internal damage has been sustained. Many probabilistic methodologies have been proposed, incorporating micromechanics,

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laminate theory, manufacturing defects, operating environment, and impact damage, but few have addressed the importance of inspection intervals and damage-detection capabilities.

The present study is based on a probabilistic damage-tolerance analysis (PDTA) with consideration of the following parameters: inspection intervals; statistical data on damages; loads; temperatures; damage-detection capability; and residual strength of the new, damaged and repaired structures. The inspection intervals are formulated based on the damaged structure's probability of failure and the quality of the structure's repair.

## Reliability Formulation

Modern damage-tolerance philosophies require that damage accumulated during the service life of a component be detected and repaired before the strength of the component is degraded beyond some design threshold. The following simple example describes some fundamental concepts of probabilistic structural analysis and design.

The assessment of the probability of failure for deterministically defined residual strength history is shown schematically in Fig. 1.

Let us assume that every structural component in an infinite fleet has this residual strength ($R$) history. The initial $R$ is 1.5. At the instant $t_0$ the damage of size $D$ occurs and $R$ is decreased to the value 1.1. At the time instant $t_1$ the damage is repaired and strength fully restored. There are three intervals $t_i$ of constant strength $S_i$. Failure occurs when the random external load exceeds the residual strength.

The probability of failure per life is expressed as:

$$P_f = 1 - \prod_{i=1}^{N=3}[1 - P_f(S_i, t_i)] \tag{1}$$

where $P_f(S_i, t_i)$ is a probability of failure per $i$th interval.

Let us assume the load exceedance curve looks like that in Fig. 2.

The cumulative distribution function (CDF) of maximum load per time ti may be written as [7]:

$$F_{l\max}(S_i, t_i) = e^{-H_t(S_i)t_i} \tag{2}$$

It is easy to find from Figs. 1 and 2 and Eq. (2) that $P_f(S_1, t_1) = P_f(S_3, t_3) = 6.1210^{-6}$, $P_f(S_2, t_2) = 4.2610^{-2}$ and the summary $P_f = 4.2610^{-2}$. In plain English, the probability of encountering a failure load while the structure is compromised by a damage event (i.e. between the occurrence of damage and its discovery and repair) is 1 in 4260.

Now, let us itemize some details of random damage events as they occur in the real world:

1) There may be any number of damages per life.

2) There may be several different types of damage (e.g., through crack, indentation, delamination, disbonding)

3) Damages occur at random times.

4) Damages have different sizes.

5) There may be several different types of inspection (preflight visual inspection, maintenance inspection, and so on).

6) Damage life depends on the frequency of inspections and the capability of inspection to detect damage.

Figure 3 shows a history of the size of random damage events over time. Two types of damage are supposed: delamination and a hole.

We need to convert the damage size into a realization of residual strength before we can calculate the probability of failure using Eq. (1). To achieve this, we need to take into account a number of deterministic and random variables discussed in the next two sections.

## Deterministic Variables or Categories

Before introducing the random variables and their functions, we must first specify the following deterministic variables, or categories: design load case/failure mode; damage type; inspection type/type of repair. All are readily derived from existing data sets in common use,

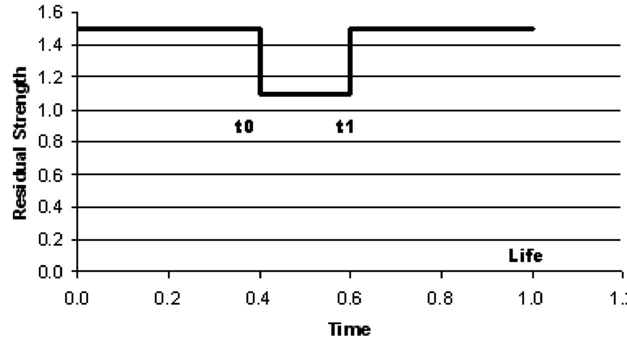


Fig. 1 Random residual strength life history.

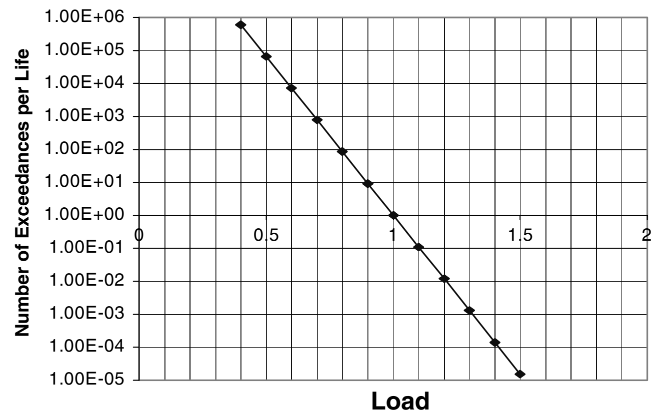


Fig. 2 Load exceedance curve.

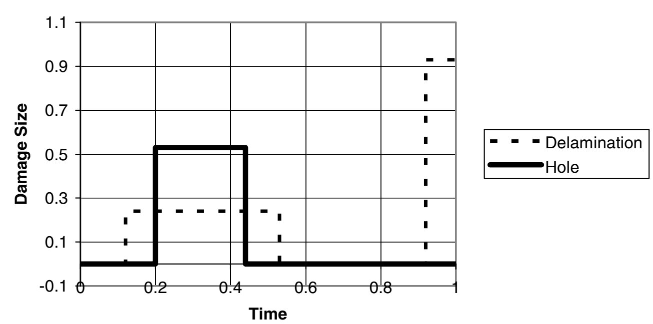


Fig. 3 Damage size lifetime history.

and minimal effort should be necessary for their identification, collection, and adaptation to the methods discussed.

### Deterministic Design Load Cases and Failure Modes

In accordance with current design practices, the strength and rigidity of an aircraft structure is analyzed for a finite set of conditions called design load cases (DLC). There may be any number of possible load cases, but for each particular substructure the engineer will select only the critical few related to the substructure's potential to fail under load conditions (bending moment, torque, shear force, and so on) anticipated to occur in various segments of the flight profile. Flap load cases, for example, would be separated into one DLC for takeoff, and another for landing.

### Deterministic Classification of Defects and Damages

As with design load cases, we need also to identify a finite set of defect and damage types (hole, delamination, surface dents, and so on) that can be classified and related to available methods for evaluating the residual strength. The classification the user chooses will depend on the characteristics and level of detail of damage data they have access to, how that data relates to calculations of residual strength, and the design load cases associated with each component that sustains damage. If a component experiences only tension stress, for example, primary attention would be paid to through damage, as delamination would be of minor importance.

### Deterministic Description of Inspection and Repair

Inspection types are distinguished by their methods, time, and frequency. Currently the time and frequency of inspection is a deterministic variable defined by the inspection schedule. The preflight type of inspection is always defined. If this inspection is not specified in the maintenance documentation, we assume their existence but at low resolution.

## Random Variables

The following nine random variables need to be considered when assessing the structural integrity of a defective or damaged component:

1) Number of damages per life ($N_D$) for each damage/defect type;
2) Time of damage initiation;
3) Damage/defect size $D$ for each damage/defect type;
4) Time $t_i$ or number of inspections from damage initiation to repair, which is a random function of damage size and inspection schedule;
5) Initial failure load (initial strength) for each design load case;
6) Average residual strength for each damage/defect size/type within each design load case;
7) Failure load of repaired structure for each damage/defect type;
8) Structural load for each design load case;
9) Structural temperatures $T$;
10) Strength/stiffness degradation due to environmental exposure.

Most of the above-mentioned variables are described either by a probability density function (PDF) or an exceedance curve, with the exception of two variables having fixed PDFs: the damage event is assumed to be a rare event and therefore the number of damages ($N_D$) is described by Poisson distribution; and the number of inspections to detect damage is described by geometric PDF.

### Probabilistic Description of Loads

As already mentioned, we describe the external loads in terms of exceedance curves and then use Eq. (2) to obtain the CDF of maximum load per certain time. In the modern practice of fatigue analysis, the life of fatigue-critical components of a structure is predicted by taking the number of cycles (time) it takes for cracks to appear in laboratory fatigue tests, and correlating it with cumulative damage in operation. This cumulative damage is predicted statistically using a load exceedance curve. It is not easy to obtain adequate statistical information (flight measurements) on the loads (stresses) for all structural sites of interest. So, cumulative damage is predicted using a probabilistic description of governing flight parameters (e.g., maneuver load factors) or a probabilistic description of atmosphere (gust), sinking speeds at landing, and so on.

These parameters are well studied and their frequency of occurrence can be easily predicted if the typical flight profiles are known. However the local structural stresses are determined by a combination of flight parameters (design load factor and speed, for example). This problem is usually solved as follows:

All flight profiles of aircraft are divided up into individual flight segments where the load distributions are approximately constant, and so the load variation for each flight segment can be described by one governing parameter. Usually the static design load cases are attributed, in a conservative manner, to these flight segments, so that the load distribution for a pull-up maneuver, for example, would be used together with the maneuver's load factor as its governing parameter, and its frequency of occurrence as determined by its load exceedance curve. Rough air loads are described by gust velocity as the governing parameter, along with the load's exceedance curve and the flight segment's corresponding load distribution (cruise, flaps-down configuration, and so on). Landing loads are described by sinking speed as the governing parameter together with the appropriate two-point landing load distribution.

The linear relationships between the governing load parameter and the stress in a considered location is determined from stress analysis for the corresponding design load case. Applying some relevant strength criterion, the occurrence of site loads can be obtained.

In addition to load exceedance curves, the proposed method and software optionally use some popular PDFs such as Gumbel extreme value type I, lognormal, and so on.

### Probabilistic Description of Structural Temperature

Temperatures and load are certainly correlated variables and in general should be described by a bidimensional PDF. However the statistical data on structural temperature are available only for supersonic aircraft, reentry vehicles, and similar. We are therefore going to use a simplified model where this correlation is hidden in different flight segments, or design load cases. Temperatures are closely related to the external loads and the way they are described may be similar. The CDF should be obtained for each flight segment or design load case. Within one design load case we will consider mechanical load and temperature to be independent random variables. All that is missing is the cumulative probability distribution (CDF) of the duration of the various temperatures, which may be derived from the aircraft's predicted design usage.

As a rough approximation the model described in [8] can also be used to obtain initial CDF. According to [8], this function consists of two branches: the first is common for all aircraft and characterizes the frequency of low temperatures, and the second depends on the maximum Mach number $M_{\max}$ of a given aircraft. The function can be approximated by the formula:

$$F(T) = 0.5 + \Phi_0\left[\frac{T - T_0}{\sigma_T}\right]$$

where $\boldsymbol{T}_0 = 310$ K; $\boldsymbol{\sigma}_T = 25$ K at $\boldsymbol{T} < \boldsymbol{T}_p$ $\boldsymbol{T}_0 = 0$ K; $\boldsymbol{\sigma}_T = 61 + 11\boldsymbol{M}_{\max}$ at $\boldsymbol{T} > \boldsymbol{T}_p$.

Here:

$$T_p = \frac{310(61 + 11M_{\max}^2)}{36 + 11M_{\max}^2}$$

$\Phi_0$ is a tabulated Gauss–Laplace function, and $\boldsymbol{T}$ is a boundary-layer temperature in degrees Kelvin.

### Probabilistic Description of Defects/Damages

Our description of defects and damages is similar to that for loads. Available statistical data, as well as relevant mathematical models derived from it, can be used for a probabilistic description of defects/damages. Defects and damages are described by their frequency of occurrence $H_D$ (similar to the load exceedance curve) for each of various damage sizes $D$, and then derived for each type of damage or by the damage size PDF plus the Poisson distribution parameter $\lambda$, which represents the average number of damages per life.

We use two classes of mechanical damages: manufacturing defects; and operational damages. The damage types within these classes are the same. Defects and damages are described by similar exceedance curves but these curves are used in different ways. Manufacturing defect is generated randomly together with initial strength in time instant $t = 0$. Operational damages are randomly scattered over the life using a uniform PDF.

The relationship between the exceedance curve $H_D(D)$ for a damage/defect size and the corresponding CDF is defined as follows:

$$F_D(D) = 1 - \frac{H_D(D)}{H_D(0)}$$

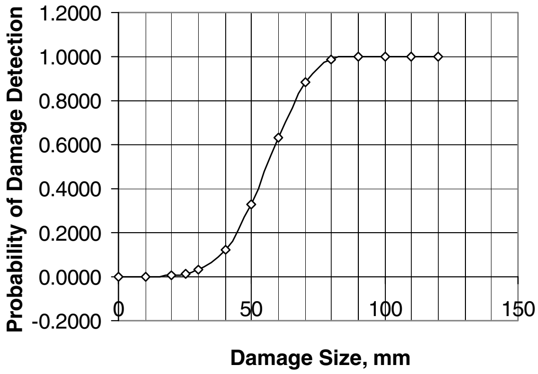


**Fig. 4 Probability of damage detection per inspection.**

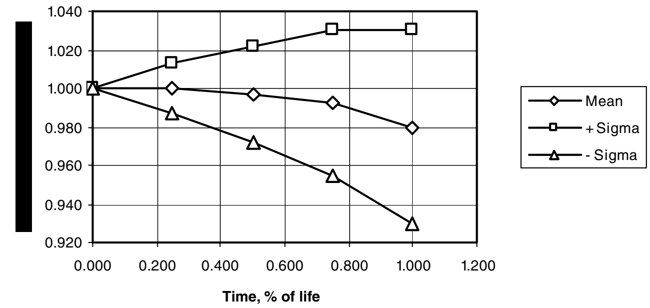


**Fig. 5 Randomized aging knockdown factor.**

#### Probabilistic Description of Inspections

The efficiency of inspection should be described by the probability of detecting the damage of a given type and size. This probability is shown in Fig. 4.

Only a few attempts to identify this function are known from the literature. Ideally special tests would be needed to obtain this probability. Having representative experts inspect, by the appropriate methods, different zones of a structure having damages of different size and type could achieve this. The probability of detection is determined as the ratio of a number of successful inspections to their total number. It is also possible to determine the detection probability by comparing the empirical probability function of detected damages with the theoretical one, and assuming that their difference is the probability for detection for various sizes of damage.

The random time to detect damage can be expressed as $t_{\mathrm{dam}} = T^{*}\xi$, where $\xi$ is a discrete random variable having a geometric distribution and $T^{*}$ is an interval between inspections. By definition, a discrete random variable $\xi$ is said to have a geometric distribution with parameter $p > 0$, if $\mathrm{Prob}[\xi = n] = p(1 - p)^{n}$.

In our application, this parameter $p$ depends on a damage size and should be determined from the data like that represented in Fig. 4.

#### Description of Repairs

There may be several decision-making algorithms if damage is detected. Examples include the following:

1) If delamination size less than 2 cm is detected in field inspection, the repair is not necessary.

2) If delamination is more than 2 cm but less than 5, the repair may be postponed until regular maintenance.

3) If delamination is more than 5 cm, it should be repaired at once.

At present, only the simplest algorithm is realized in a model. When damage is detected there are the following two options: repair now or postpone until scheduled maintenance. The method of repair is directly determined by the type of inspection that resulted in its detection. The degree to which the original strength is recovered is determined by the method of repair corresponding to the inspection. The degree of strength recovery is described as follows:

Assume that we made a number of composite components of similar design. We then simulate damages of equal type, size, and location in each of them, repair those damages with the chosen method, test the components, and obtain an average failure load and failure load variance. We will describe the strength after repair by two parameters: the "strength recovery coefficient" (which is equal to the ratio of the average failure load to that of a new structure); and the coefficient of variation of the strength after repair.

It was found that this logic does not strongly influence the final result. However it is possible to devise the logic that considerably influences the probability of failure (POF).

#### Description of Strength/Stiffness Degradation due to Environmental Exposure

The possibility of including the available probabilistic water diffusion model [9] into this software has been discussed in [10]. The model in [9] is based on Fick's law of diffusion simulation. As the integration is rather time consuming, it has been decided to use this as a separate software branch. The integral effects of aging and water absorption are simulated as quasi-random strength knockdown factor versus time of operation, as shown in Fig. 5.

Rather than intervals of constant residual strength, this results in a linearly changing residual strength in its time history. Fortunately it can be shown that for most continuous load PDFs, the probability that the load exceeds the linearly changing strength is equal to that for a constant strength equal to the mean value between the ends of time interval. We have used this in our POF calculations outlined below.

## Computational Methods

As was mentioned earlier, the problem of determining the reliability of damage-tolerant composite structures is addressed here with computer simulation. Several sampling-based probabilistic methods have been considered as candidates, such as the standard Monte Carlo method (MC) and various methods based on fast probability integration (FPI), as in NESSUS. The latter includes first and second order reliability methods (FORM, SORM), various importance sampling methods and advanced mean-based methods. It was found that most popular FPI methods work well with smooth performance $g$ functions, but are inappropriate in tasks involving discrete variables. As the model considered here uses some discrete variables and variables with truncated PDF, main attention has been paid to MC simulation. It is well known that although MC provides accurate results, it is time consuming, especially if the $g$ function is evaluated through a finite-element model with many degrees of freedom. Fortunately, the MC simulation realized for our model will compute, on a typical office PC, a reasonable POF level from $10^{-3}$ to $10^{-5}$ per life in less than 10 s. This is generally enough for parametric analyses.

Two closely related methods have been developed depending on the realization of $g$ function: the strength-load method (S–L); and the POF. Both methods include simulated residual strength histories (RSH), consisting of a sequence of intervals with constant or linearly changing strength. In the S–L method a maximum random load and random temperature are generated for each interval. The load is compared with the residual strength computed depending on temperature and aging. When L > S, the failure is recorded. The ratio of the number of structural lives with failure to the total number of simulated lives is the POF. In the POF method, the probability of failure is calculated as in Eq. (1). Many such POFs are sampled, and the average value and standard deviation are calculated. A number of sampled histories is selected to satisfy the accuracy requirements. This latter method reduces the size of the analysis region and generally is significantly more efficient than S–L simulation. The advantage of the S–L method is that a description of every failure is recorded. If the model is complicated by many design load cases, damage types, and inspection types, this feature may help to shed light on the situation better than sensitivity analysis.

#### Residual Strength History Simulation

Each RSH is based on a number of intervals with damage of a constant size. The starting time of each interval is a random value and the duration of each interval is a random function of the probability of

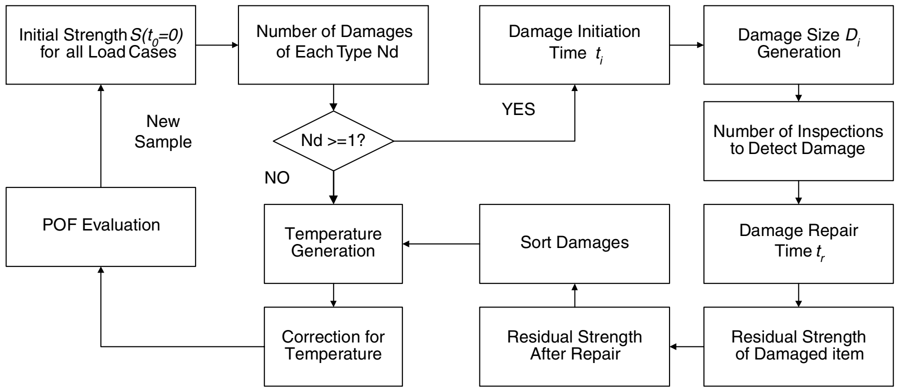


**Fig. 6 Simulation flowchart.**

damage-detection and inspection interval. The history may be randomly simulated using a finite set of random variables such as damage occurrence rate and probability of damage detection.

So, we need to create a generator of random functions characterized by a multivariate PDF, $f(v)$. This generation scheme is illustrated in Fig. 6.

The sequence for the generation of one residual strength history is as follows:

1) For all considered design load cases, generate initial strength values, $S(t_0 = 0)$.

2) Generate the number of defects/damages for each considered damage/defect type (if any).

3) If no manufacturing defects and operational damages occur during this life, generate, for all considered design load cases, random temperatures of the structure at each instant of maximum load occurrence.

4) Correct the initial strength values $S(t_0 = 0)$ for temperature and aging, and evaluate the lifetime POF. Start a new generation cycle.

5) If the total number of defects and damages is more than zero, generate sizes for all manufacturing defects of all types, then generate residual strength values $S(t_i)$ for all design load cases. The manufacturing defects are recorded and then treated as operational damages occurring at $t_0 = 0$.

6) Scatter operational damages over the life using a uniform distribution generator. Generate time instants $t_{\text{in}}$ for each and add to the list of damages.

7) Generate values for damage size.

8) For each damage, randomly generate the time of detection and time of repair using the probability of detection and the inspection schedule. Generate a random number of inspections for different types of damage detection using a geometric PDF, and then correlate these to the inspection schedule. Find the minimum time of detection from randomly simulated times for different types of inspection. Determine repair time instants and the duration of damage existence.

9) Generate values of residual strength $S(t_i)$ with damage, and account for aging by calculating the mean value between the ends of the interval.

10) As a result of previous actions we have a set of intervals with strength $S(t_i)$. We may have intact structure intervals, overlapping intervals of damaged states, and repaired state intervals. Here a special procedure is applied to sort the intervals and eliminate possible overlapping. This procedure results in a new sequence of intervals, characterized by length $t_i$ and level of residual strength $S_i$. The sequence also includes intact structure intervals and repaired structure intervals (if any).

11) Correct residual strength values, $S_i$, for temperature.

12) Evaluate $P_f$ using Eqs. (1) and (2). Start a new generation cycle.

## Demonstration Cases

### Example Problem 1

The input data we use for this problem are typical of those for current damage-tolerant composite aircraft parts. The structure used complies with the requirements of AC 20-107A. The main assumptions are listed below:

1) We assume just one design load case. The load exceedance curve per life is expressed as

$$H_t(x) = H_0 \exp\left(\frac{-x}{b}\right)$$

where $H_0 = 4.2683 \times 10^9$ and $b = 0.112742$. These parameters are based on observations published by Taylor [11]. The frequency of 1 g level crossings is 60 times per flight hour, and the frequency of limit 2.5 g level crossings is one per life (10,000 flights).

2) We assume only one type of damage. It is known that the size of dust and sand particles, hailstones, and raindrops in the atmosphere generally follows an exponential distribution, and so we assume an exponential distribution of initial impact damage size. The damage exceedance curve will have the simple shape described by the following equation:

$$H_D(D) = E_0 e^{-D/B} \tag{3}$$

If we take two points on the damage exceedance curve, $P_1(D_1)$ and $P_2(D_2)$, corresponding to damages that are in the region of 100% detected, and assume that exceedances of the initial and detected damages are equal, we can get the following expressions:

$$e^{-(D_1-D_2)/B} = \frac{P_1}{P_2}; \qquad B = \frac{D_2 - D_1}{\ln(P_1/P_2)}$$

Substituting the data obtained by Lin [5], we can obtain $B =$ 1.6–1.8 in. In this example $B = 1.5$ in. and $E_0 = 1$ (one damage event per life on average) are used.

3) We assume here just one type of inspection. The probability of damage detection is described by the Weibull function [4]:

$$P_D(D) = 1 - \exp\left(-\frac{D}{\beta}\right)^{\alpha}; \qquad \alpha = 1.4; \qquad \beta = 1.64 \text{ in.}$$

4) The initial average value of strength is equal to $2.5 \times 1.5 \times 1.415 = 5.3$. Here 2.5 is the limit load factor, and 1.5 is a factor of safety. The last factor is an additional margin of safety used for composite structures in a considered design load case. The strength scatter is described by a Gauss PDF with the coefficient of variation, $C_V = 5\%$, that is the same for both the initial strength and the strength of the damaged structure. The reduction of residual strength in relation to the damage size $D$ is described by the following function:

$$Y(D) = A + (1 - A)\exp\left(-\frac{D}{G}\right); \qquad A = 0.55; \qquad G = 2.0 \tag{4}$$

[where $A$ is the residual strength asymptote, and $G$ is the residual strength slope]. All requirements of AC 20-107A are satisfied.

5) The strength after repair is described by a Gauss PDF with the average equal to 100% of initial strength and the coefficient of variation, $C_V = 5\%$.

6) The strength is considered to be independent of temperature.

**Validation of Example Problem 1**

To validate the method, we compared the exact solution with the NESSUS solution for a simplified task. The results are shown in Fig. 7.

We compared the simulation results with the following numerical integration:

$$\text{POF} = \int_0^\infty f_{L\,\max}(x) F_S(x)\,\mathrm{d}x \tag{5}$$

where $f_{L\,\max}$ is a PDF of maximum load per service life and $F_S$ is a CDF of structural strength. Here we assumed that there are no damages or any other strength degradations. Various shapes and parameters of those functions were tested and good agreement with both the L-S and the POF methods was obtained.

The NESSUS model we obtained has the following features:

1) Exactly one damage per life, occurring in the middle of life;

2) Random loads for undamaged, damaged, and repaired state are sampled from a Gumbel distribution;

3) Random initial strength is sampled from a normal distribution;

4) Random damage size $D$ is sampled from an exponential distribution;

5) Random inspection interval is sampled from a normal distribution with $C_V = 10\%$

Figure 8 shows a good agreement between the two considered methods for the probability of failure using the above-mentioned input data.

**Parametric Study of Example Problem 1**

In general, the POF and the corresponding inspection interval depend on all input parameters. The most important have been selected for this particular case. In all cases the inspection interval has been determined as one corresponding to a probability of failure of about $10^{-4}$ per life.

*Effect of Various Extensions of the Load Exceedance Curve*

The PDF for maximum load is determined by an exceedance curve of external loads per flight hour. The most reliable method for deriving the load exceedance curve is to measure actual loads in flight. The exceedance curve is usually known for the range between the loads experienced in level flight to those at the limit load factors. For the purpose of reliability assessment, the reasonable extrapolation of this curve to the region between the limit load and the ultimate load is needed. Figure 9 shows three possible extensions of the load exceedance curve and Fig. 10 shows the effect of those extensions. Extension 1 is suitable if the load source is atmospheric turbulence; extension 3 is more suitable for maneuver load when it is essentially restricted, for example, aileron deflection; extension 2 may be good for a combination of 1 and 3.

*Effect of Damage Exceedance Curve Variations*

The damage exceedance curve in this example is described by the exponential function (3). The effect of the damage exceedance intercept $E_0$ is shown in Fig. 11. The effect of damage exceedance slope is shown in Fig. 12.

*Effect of Average Detected Damage*

In this example, the probability of damage detection is described by the Weibull function. The influence of the Weibull scale parameter is shown in Fig. 13.

*Effect of Residual Strength vs Damage Size Variations*

To describe residual strength vs damage size, we use Eq. (4). The effect of the residual strength asymptote $A$ is shown in Fig. 14.

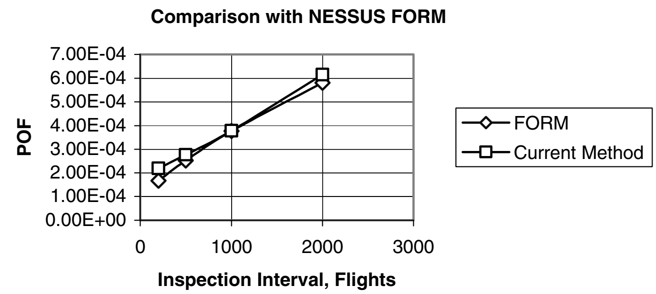


Fig. 7 Comparison with NESSUS.

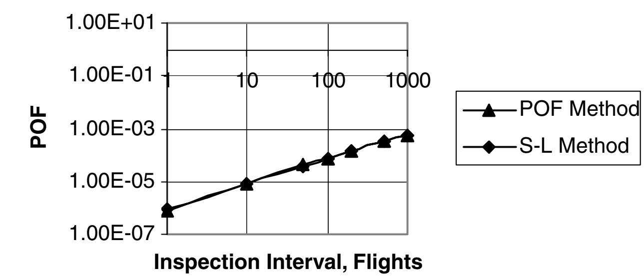


Fig. 8 Comparison of POF and S–L methods.

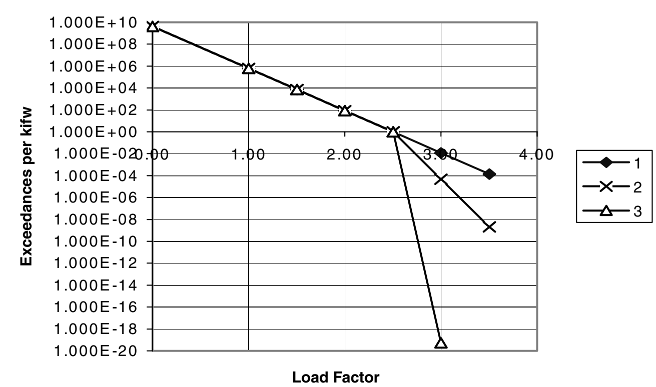


Fig. 9 Various extensions of the load exceedance curve.

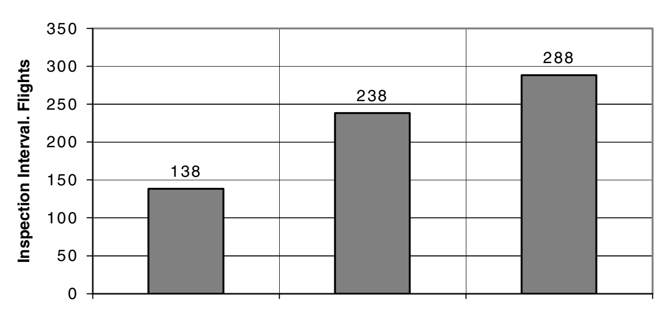


Fig. 10 Effect of various extensions of load exceedance curve.

The effect of the residual strength slope $G$ is shown in Fig. 15.

*Effect of Strength Recovery After Repair*

Strength recovery characterizes the quality of repair, and the quality of repair depends only on the method of repair. But the probability of failure depends, generally, on the percentage of strength recovery. The higher this percentage, the lower the POF


Fig. 11 Effect of damage exceedance intercept.

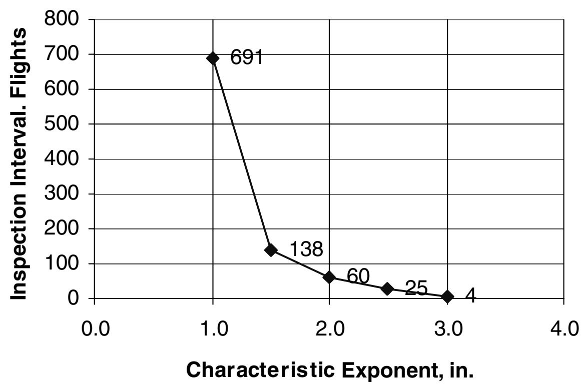


Fig. 12 Effect of damage exceedance slope.

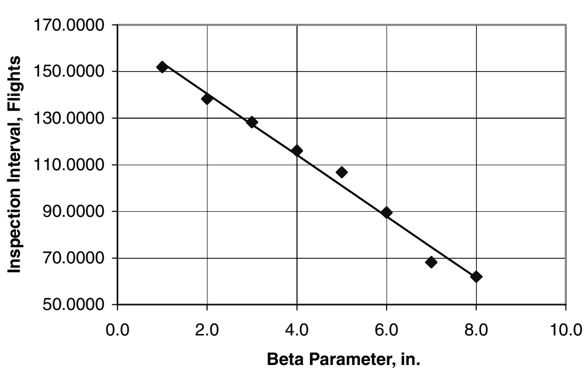


Fig. 13 Effect of average detected damage.

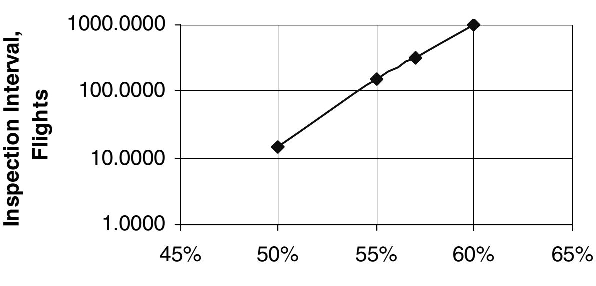


Fig. 14 Effect of residual strength asymptote.

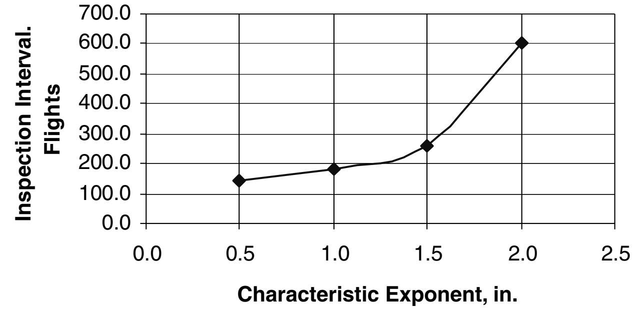


Fig. 15 Effect of residual strength slope.

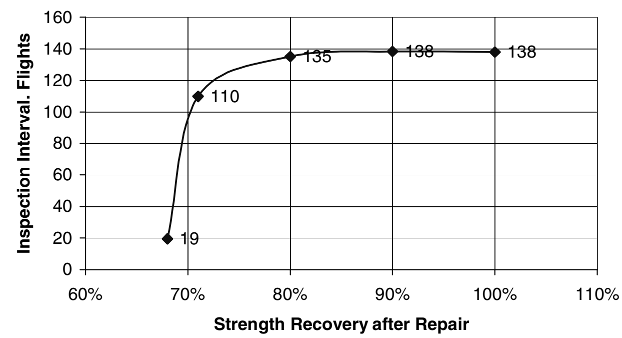


Fig. 16 Effect of strength recovery.

should be. Strength recovery after repair is shown graphically in Fig. 16.

We determine the inspection interval assuming a constant level of POF $= 10^{-4}$ per life, or $10^{-8}$ per flight. If we have a perfect repair method (strength recovery $= 100\%$), we obtain a POF $= 10^{-4}$ per life, with an inspection interval equal to 130 flights. If our repair method restores only 80% of the initial strength, we have to inspect the structure more frequently to obtain the same POF $= 10^{-4}$ per life. But if our repair restores only 65% of the initial strength, we cannot reach POF $= 10^{-4}$ per life, even if we inspect after each flight.

Assume that we repair some damage and then test the structure, obtaining a failure load of only 90% of its initial strength. We might conclude that our repair method was inferior, as it increases the probability of failure. But our calculations of POF show that this is not the case. Rather, our calculations show that this structure is insensitive to the quality of repair. In fact, the quality of repair influences the POF and inspection interval only when the strength recovery is less than 80%.

We conclude that the "inferior" repair method is adequate. This is not trivial and is really a quite important conclusion. It means that the quality of repair may not be as important as we suppose. Obviously this is not a general rule and depends mostly on the behavior of the residual strength vs damage size but, at least in some cases, resources need not necessarily be expended on the invention and application of "superior" repair methods.

**Example Problem 2: Composite Fin**

The input data for this problem were taken from FAA reports [6,12,13]. This example works quite well for comparing the S–L method with the POF method.

The main features of the problem are

1) The damage rate is quite low (0.231 per life).

2) The load cases include one subsonic and one supersonic case with elevated temperatures.

3) Two types of damage are considered: delamination and hole or crack.

4) Two types of inspection are considered: pre/post flight (type 1) and a special (type 2) inspection method applied during maintenance. The detection probabilities are shown in Fig. 17.

The analysis shows that most trends are similar to those for the previous example. It is worth mentioning that the POF for the second type of inspection is almost independent of the inspection interval because the majority of damages are detected with the type 1

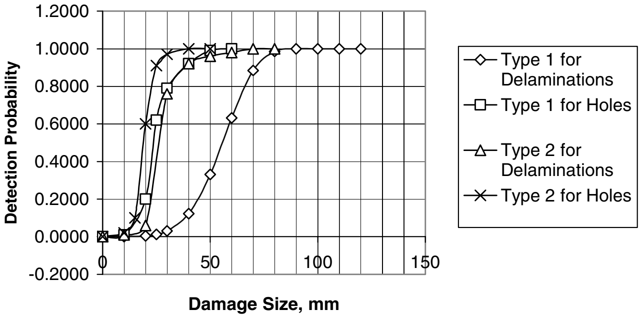


**Fig. 17 Detection probability vs damage size.**

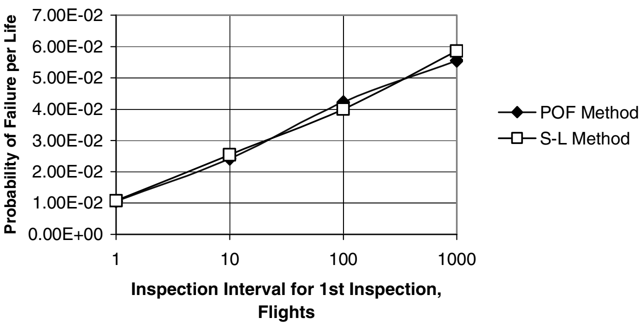


**Fig. 18 POF vs interval comparison.**

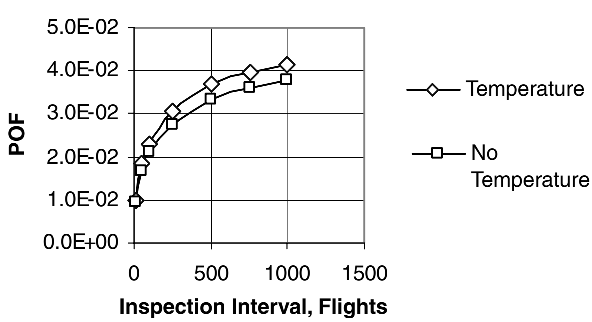


**Fig. 19 Effect of temperature on POF.**

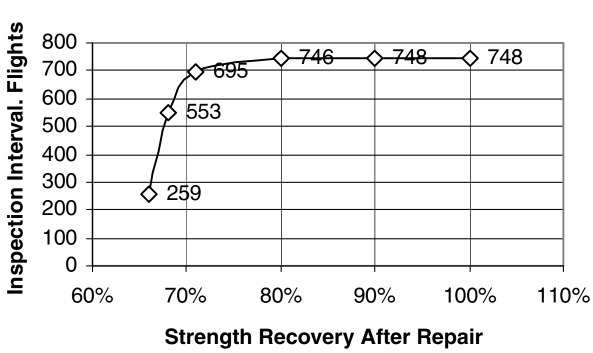


**Fig. 20 Effect of strength recovery after repair.**

inspection. Some specific results of parametric analysis are shown in Figs. 18–20. The POF we obtained is rather high for this composite fin. We can expect about 1% of fin composite panels to fail within their usage life. The designer agreed to pay this tribute to mechanical damages, however, because it was supposed that the failure would not lead to a catastrophic structural failure. The fin has two metal spars that carry an essential part of external load. This aircraft has two main fins and two additional dorsal fins that guarantee the completion of a successful flight even if one composite panel should fail.

## Conclusions

In this paper we proposed a method for quantifying the damage tolerance and reliability of aircraft composite structures in the presence of multiple uncertainties. The method's main purpose is to discover the optimum inspection schedule while maintaining a high standard of structural reliability. The effectiveness of the method is illustrated for two typical aircraft composite structure models.

In summary, we can make the following conclusions:

1) Probabilistic methods may be used to quantify the reliability of damage-tolerant composite aircraft structures, and to establish optimum inspection intervals, enabling aircraft manufacturers, operators, and flight certification authorities to establish maintenance and service guidelines that reduce life-cycle cost.

2) The inspection interval that ensures a reasonably high reliability depends primarily on statistical characteristics of external loads, damage rate, and the residual strength of the damaged structure.

3) The behavior of the load exceedance curve near the limit and higher, or the scatter of maximum load per life, have large influences on reliability and inspection intervals and should be a subject of special attention when applying the probabilistic methods.

4) Because of the lack of statistical data, the most uncertain variables in any probabilistic damage-tolerance design method are damage size and frequency. Unfortunately, available in-service data may not contain complete descriptions of damage size and frequency, their structural locations, or the inspection method used.

5) The extent of strength recovery after repair does not, in a broad range of values, significantly influence the reliability and inspection interval.

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